CS 6505: Algorithm, Computability and Complexity

Fall 2007

Sample Midterm Questions

Question 1

Turing-recognizable languages are closed under complementation. That is, if language L is Turing-recognizable, then so is its complement $\overline{L}$.

ANSWER: FALSE.

JUSTIFICATION: In class it was shown that the language A_{TM} is Turing-recognizable but its complement is not Turing-recognizable. Thus the statement is false.

Question 2

If the capacities of edges in a flow network G are changed and the total increase (resp. decrease) of all edge capacities is k, then the value of a maximum flow of G increases (resp. decreases) by at most k.

ANSWER: TRUE.

JUSTIFICATION: Since the total capacity increase (resp. decrease) is k, the capacity of a minimum cut increases (resp. decreases) by at most k. By the Max-Flow Min-Cut Theorem, the value of a maximum flow equals the capacity of a minimum cut. Therefore the value of a maximum flow increases (resp. decreases) by at most k.

Question 3

If the capacity of each edge in a flow network $G = (V, E)$ is multiplied by k, then the value of a maximum flow in G is also multiplied by k. Moreover, if f is a maximum flow in the original network (before the edge capacities are changed), then f' defined as $f'(e) = kf(e)$ for each edge $e \in E$ is a maximum flow in the new network (after the edge capacities are changed).

ANSWER: TRUE.

JUSTIFICATION: Let G' denote the flow network after the edge capacities are multiplied by k. It can be easily seen that for every flow f of G, $f' = kf$ is a flow of G' (verify that the capacity constraint and flow conservation hold), and conversely for every flow f' of G', $f = \frac{1}{k} \cdot f$ is a flow of G. Thus the map $f \mapsto cf$ gives a one-to-one correpondence between flows of G and flows of G'. Hence if f is a maximum flow of G, then $f' = kf$ is a maximum flow of G' and $|f'| = k|f|$.

Question 4

Although the original Ford-Fulkerson algorithm may fail to terminate in the case where edge capacities are arbitrary real numbers, it can be slightly modified so that it is guaranteed to terminate in the case where edge capacities are rational numbers, regardless of how the augmenting paths are chosen. This is an immediate consequence of the statement in Question 3.

ANSWER: TRUE.

JUSTIFICATION: Let $G = (V, E)$ be a flow with rational capacities. The following modification of Ford-Fulkerson computes a maximum flow of G:

1. Let k be a common multiple of the denominators of all edge capacities. Multiply the capacity of each edge in G by k, and denote by G' the resulting flow network.

2. Use Ford-Fulkerson to find a maximum flow f' of G'.

3. Output $f = \frac{1}{k} \cdot f'$.

Since G' has integer-valued capacites, Ford-Fulkerson is guaranteed to terminate and output a maximum flow f' of G'. By Question 3, $f = \frac{1}{k} \cdot f'$ is a maximum flow of G.